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Calculation of plastic strain ratio from the texture of cubic metal sheet

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A simple method is presented for calculating the plastic strain ratio from crystallographic texture based on ideal sheet textures. The *R* values for several f c c and b c c metals were calculated as a function of angle to the rolling direction using this method. The agreement between calculated and measured *R* values was satisfactory.

1. Introduction

Deep drawability is closely related to the plastic strain ratio or *R* value, which is defined as the ratio of true strains in the width and thickness directions under tension. For planar isotropic sheets, a higher *R* value implies higher resistance to thinning in the thickness direction, resulting in a higher limiting draw ratio. For planar anisotropic sheets, the *R* value varies with the tensile direction. The variation of *R* value with tensile direction is associated with earing behaviour in deep drawing. Earing occurs along the directions of higher *R* values.

It is well known that sheet anisotropy is closely related to sheet texture. A method has been proposed by Hosford and Backofen [1] for the prediction of *R* values and yield stress as a function of sheet texture based on finding the combination of slip systems that minimizes the work per unit volume required to produce a given strain. Vieth and Whiteley [2] and Fukuda [3] proposed very simplified methods for the prediction of *R* values in relation to sheet texture.

In these models it was assumed that the sheet was made up of texture components having ideal crystallographic orientations; that the texture components contributed independently to the overall deformation of the sheet; and that the effectiveness with which a particular texture component contributes to the *R* value was determined by the volume fraction of that texture component and by the relative favourability of the orientation of the slip systems operating in the various texture components present. However, the choice of operating slip systems and their contributions to the *R* value calculation were different in Vieth and Whiteley [2] and Fukuda [3]. Their calculated results for the *R* value generally ranged from zero to infinity. Perovic and Karastojkovic [4] used the Vieth–Whiteley–Fukuda equation for the *R* value calculation, but they took at least two active slip systems with different slip planes for which the values of the Schmid factor were greatest.

Lee [5] advanced a modified form of the Vieth–Whiteley–Fukuda equation on the assumption that all the slip systems contribute to the deformation in proportion to their Schmid factors. The method gave satisfactory results for sheets whose texture might be approximated as ideal. When a sheet was composed of more than one ideal texture, the *R* value of the sheet was calculated by

$$R = \sum R_i V_i$$

where R_i and V_i are the value and volume fraction of texture component *i*. The *R* values calculated by this method were generally in good agreement with those measured. However, we found that this method gave rise to an *R* value of infinity instead of unity for a specimen with randomly distributed grains (random orientation). The purpose of this paper is to propose a more reasonable method of calculation of the *R* value for a complex texture.

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2. Planar anisotropy of the R value

Vieth and Whiteley [2] and Fukuda [3] derived the following relation for the plastic strain ratio or R value:

$$R = \frac{(\mathbf{b} \cdot \mathbf{p})(\mathbf{b} \cdot \mathbf{d})}{(\mathbf{t} \cdot \mathbf{p})(\mathbf{t} \cdot \mathbf{d})} \tag{1}$$

where vectors **b**, **t**, **d** and **p** are unit vectors along the width, thickness and slip directions and the direction normal to the slip plane, respectively. In order for Equation 1 to be used, the slip systems operating should be selected. As mentioned in the previous section, Vieth and Whiteley [2] took the slip system of largest Schmid factor from the applied tensile stress. However, Fukuda pointed out that the slip system with the largest Schmid factor never operates alone but co-operates with several other systems in order of the magnitude of the Schmid factors from the tensile direction, and calculated the R values from five slip systems using Equation 1. The calculated R values were weighted differently based on the magnitude of the Schmid factors and were averaged to give a resultant R value.

The slip systems having the larger Schmid factors will operate more easily. The slip system with largest Schmid factor does not always operate in tension of polycrystalline metals, but has the highest probability of operating, and so on. Lee [5] assumed that all the slip systems contributed to the deformation but that their contributions were proportional to their Schmid factors. Thus the following equation was proposed for calculation of the R value:

$$R = \frac{\varepsilon_w}{\varepsilon_t} = \frac{\sum [|(\mathbf{b} \cdot \mathbf{p})(\mathbf{b} \cdot \mathbf{d})| S]}{\sum [|(\mathbf{t} \cdot \mathbf{p})(\mathbf{t} \cdot \mathbf{d})| S]} \tag{2}$$

where Σ indicates the summation of all operating slip systems and S is the Schmid factor of corresponding slip systems, which can be written

$$S = |(\mathbf{1} \cdot \mathbf{p})(\mathbf{1} \cdot \mathbf{d})| \tag{3}$$

Here **1** is the unit vector along the tensile direction.

A sheet having texture $(R_1 R_2 R_3)[A_1 A_2 A_3]$ is now considered. For convenience of calculation, any direction is expressed by unit vectors. Unit vectors along the normal to the plane $(R_1 R_2 R_3)$ and the direction $[A_1 A_2 A_3]$ may be written

$$\mathbf{r} = [r_1, r_2, r_3] = [R_1/|R|, R_2/|R|, R_3/|R|] \tag{4a}$$

where $|R| = (R_1^2 + R_2^2 + R_3^2)^{1/2}$ and

$$\mathbf{a} = [a_1, a_2, a_3] = [A_1/|A|, A_2/|A|, A_3/|A|] \tag{4b}$$

where $|A| = (A_1^2 + A_2^2 + A_3^2)^{1/2}$.

Let the angle between the rolling direction **a** and the tensile direction **1** be α, then unit vectors **1**, **t** and **b** can be obtained from

$$\begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} a_1 & a_2 & a_3 \\ r_1 & r_2 & r_3 \\ n_1 & n_2 & n_3 \end{bmatrix} = \begin{bmatrix} 1_1 & 1_2 & 1_3 \\ t_1 & t_2 & t_3 \\ b_1 & b_2 & b_3 \end{bmatrix} = \begin{bmatrix} \mathbf{1} \\ \mathbf{t} \\ \mathbf{b} \end{bmatrix} \tag{5}$$

where $\mathbf{n} = [n_1, n_2, n_3]$ is the unit vector normal to both **a** and **r**, that is,

$$\mathbf{n} = \mathbf{a} \times \mathbf{r} = [a_2 r_3 - a_3 r_1, a_3 r_1 - a_1 r_3, \times a_1 r_2 - a_2 r_1]$$

From Equation 5, vectors **1**, **t** and **b** may be expressed as:

$$\begin{aligned} \mathbf{1} &= [1_1, 1_2, 1_3] \\ &= [a_1 \cos\alpha + (a_2 r_3 - a_3 r_2)\sin\alpha, a_2\cos\alpha \\ &\quad + (a_3 r_1 - a_1 r_3)\sin\alpha, a_3\cos\alpha \\ &\quad + (a_1 r_2 - a_2 r_1)\sin\alpha] \\ \mathbf{t} &= [t_1, t_2, t_3] = [r_1, r_2, r_3] \\ \mathbf{b} &= [b_1, b_2, b_3] \\ &= [-a_1\sin\alpha + (a_2 r_3 - a_3 r_2)\cos\alpha, \\ &\quad - a_2\sin\alpha \\ &\quad + (a_3 r_1 - a_1 r_3)\cos\alpha, -a_3\sin\alpha \\ &\quad + (a_1 r_2 - a_2 r_1)\cos\alpha] \end{aligned} \tag{6}$$

For a slip system $(P_1 P_2 P_3)[D_1 D_2 D_3]$, the unit vectors **p** and **d** in Equation 2 may be expressed as

$$\mathbf{p} = [p_1, p_2, p_3] = [P_1/|P|, P_2/|P|, P_3/|P|] \tag{7a}$$

where $|P| = (P_1^2 + P_2^2 + P_3^2)^{1/2}$ and

$$\mathbf{d} = [d_1, d_2, d_3] = [D_1/|D|, D_2/|D|, D_3/|D|] \tag{7b}$$

where $|D| = (D_1^2 + D_2^2 + D_3^2)^{1/2}$.

TABLE I Calculated R_{random} at various angular intervals

Angular interval (deg)	10	9	6	5	3	2	1
R_{random}	0.9962	0.9981	0.9971	0.9980	0.9985	0.9988	0.9994

Thus, if a sheet texture $(R_1 R_2 R_3)$ $[A_1 A_2 A_3]$ and a slip system $(P_1 P_2 P_3)$ $[D_1 D_2 D_3]$ are known, the R value of Equation 2 can be evaluated using Equations 4 to 7.

A sheet specimen composed of more than one ideal texture is considered. The strains in the width and thickness directions of the specimen under tension, ε_w and ε_t, may reasonably be assumed to be determined by the volume fraction of each texture component. That is,

$$\varepsilon_w = \sum \varepsilon_{wi} V_i = \int_V \varepsilon_w(g) f(g)\, dg$$

$$\varepsilon_t = \sum \varepsilon_{ti} V_i = \int_V \varepsilon_t(g) f(g)\, dg \quad (8)$$

where ε_{wi}, ε_{ti} and V_i are respectively the strains in the width and thickness directions and the volume fraction of texture component i; g and $f(g)$ are the orientation and the orientation distribution function. Therefore the R value can be written as

$$R = \frac{\varepsilon_w}{\varepsilon_t} = \frac{\sum \varepsilon_{wi} V_i}{\sum \varepsilon_{ti} V_i} = \frac{\int_V \varepsilon_w(g) f(g)\, dg}{\int_V \varepsilon_t(g) f(g)\, dg} \quad (9)$$

When the texture of component i is known, ε_{wi} and ε_{ti} can be calculated as explained in the calculation of ε_w and ε_t. When the orientation distribution function is not known, a rough estimate of the volume fraction V_i can be made on the basis of peak intensity in the pole figure. When the intensities from a family of reflection planes for a given ideal texture are not the same, the lower intensities can be assumed to come from a single ideal texture and the higher intensities are assumed to be obtained by superposition of intensities from more than one ideal texture.

3. The *R* values of a specimen with randomly distributed grains

For randomly distributed grains, the orientation distribution function $f(g)$ in Equation 9 becomes unity. Therefore the R value of the specimen, R_{random}, is given by

$$R_{random} = \frac{\int_V \varepsilon_w(g)\, dg}{\int_V \varepsilon_t(g)\, dg} \quad (10)$$

where dg may be expressed as a function of Euler angles ψ_1, ϕ and ψ_2, that is,

$$dg = \frac{1}{8\pi^2} \sin\phi \, d\psi_1 \, d\phi \, d\psi_2$$

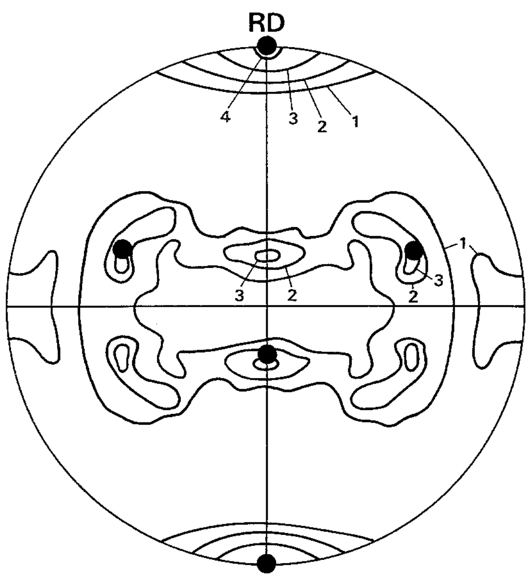

Figure 1 (1 1 1) pole figure of a copper sheet [4].

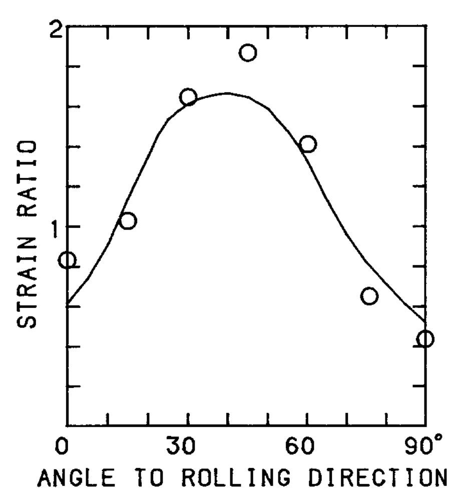

Figure 2 R value as a function of tensile direction for the specimen in Fig. 1: o, measured [4]; —, calculated based on the texture (1 1 2) [$\bar{1}$ $\bar{1}$ 1].

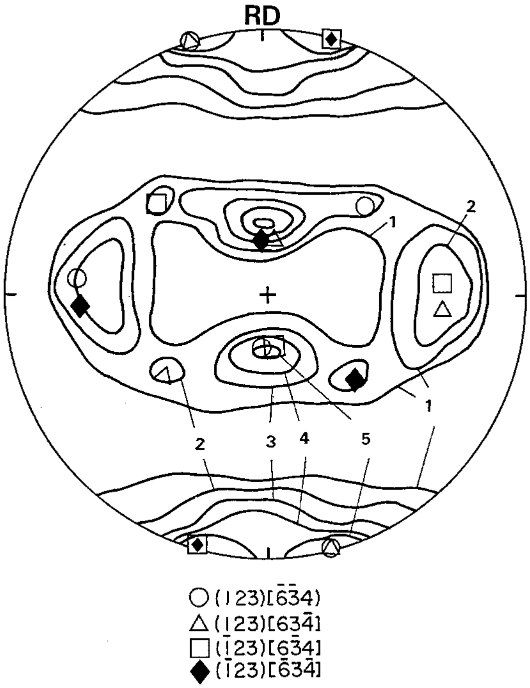

Figure 3 (1 1 1) pole figure of a 304 stainless-steel sheet [6].

For cubic metals the cubic symmetry makes the integral intervals reduce to the following:

$$0 \leqslant \psi_1, \phi, \psi_2 \leqslant \pi/2$$

Since ε_w and ε_t in Equation 9 or 10 are calculated when the orientation is given in the form $(R_1\, R_2\, R_3)\,[A_1\, A_2\, A_3]$, the orientation $g(\psi_1, \phi, \psi_2)$ should be transformed to the form $(R_1\, R_2\, R_3)$

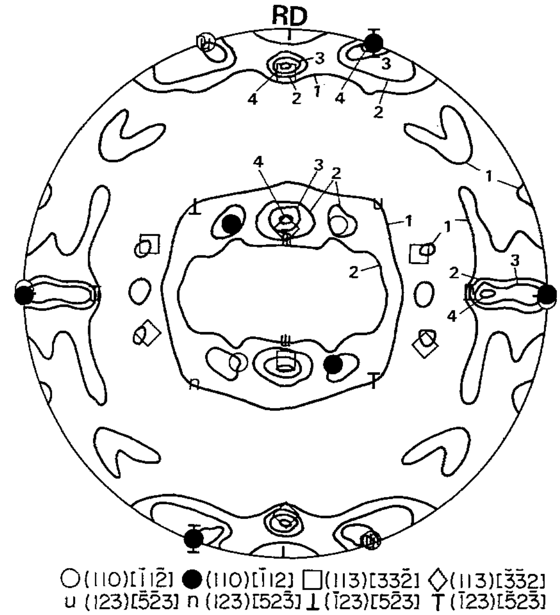

Figure 5 (1 1 1) pole figure of a brass sheet [4].

$[A_1\, A_2\, A_3]$ using the following relations:

$$\begin{aligned}
R_1 &= \sin\psi_2 \sin\phi \\
R_2 &= \cos\psi_2 \sin\phi \\
R_3 &= \cos\phi \\
A_1 &= \cos\psi_1 \cos\psi_2 - \sin\psi_1 \sin\psi_2 \cos\phi \\
A_2 &= -\cos\psi_1 \sin\psi_2 - \sin\psi_1 \cos\psi_2 \cos\phi \\
A_3 &= \sin\psi_1 \sin\phi
\end{aligned}$$

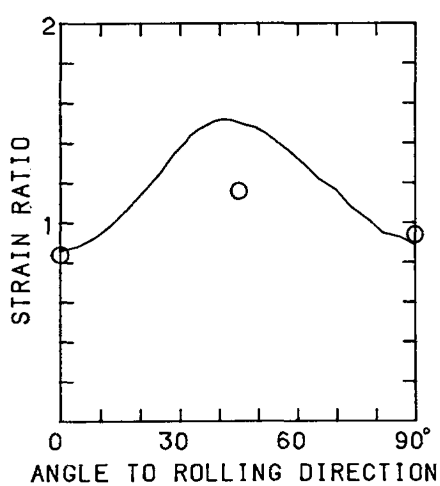

Figure 4 R value as a function of tensile direction for the specimen in Fig. 3: ○, measured [6]; —, calculated based on the texture $\frac{1}{2}(1\,2\,3)\,[\bar{6}\,\bar{3}\,4] + \frac{1}{2}(\bar{1}\,2\,3)\,[6\,\bar{3}\,4]$.

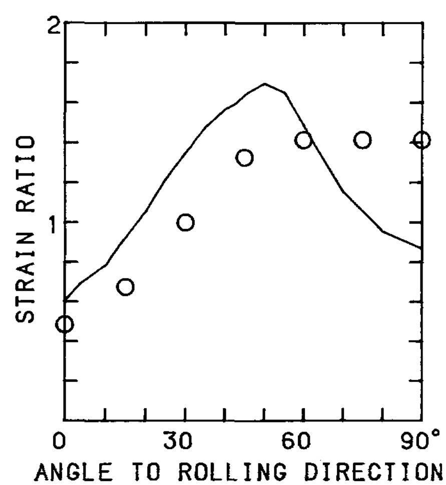

Figure 6 R value as a function of tensile direction for the specimen in Fig. 5: ○, measured [4]; —, calculated based on the texture $\frac{1}{6}(1\,1\,0)\,[\bar{1}\,1\,\bar{2}] + \frac{1}{6}(1\,1\,0)\,[\bar{1}\,1\,2] + \frac{1}{3}(1\,1\,3)\,[3\,3\,\bar{2}] + \frac{1}{6}(1\,2\,3)\,[\bar{5}\,\bar{2}\,3] + \frac{1}{6}(\bar{1}\,2\,3)\,[5\,\bar{2}\,3]$.

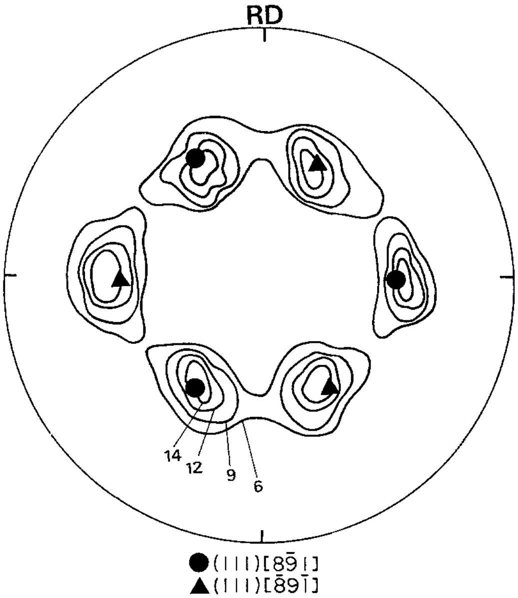

Figure 7 (0 0 1) pole figure of a 444 stainless-steel sheet [7].

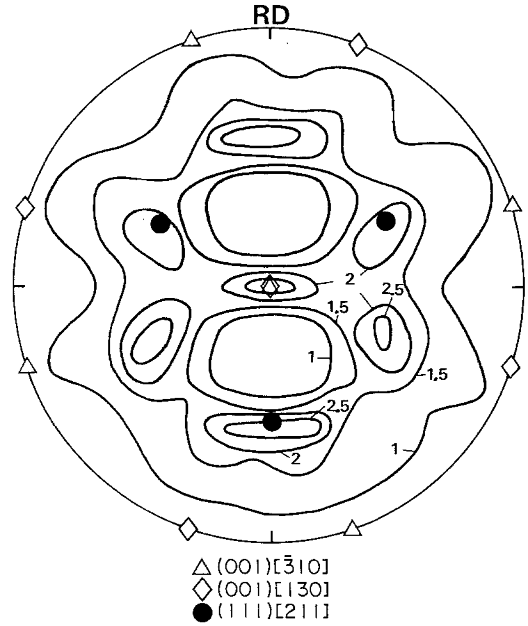

Figure 9 (0 0 1) pole figure of a 17% chromium stainless-steel sheet [8].

Numerical calculation of Equation 10 has been performed at angular intervals, 1°, 2°, 3°, 5°, 6°, 9° and 10°, and the results are given in Table I, indicating R_{random} to be unity.

4. The *R* values of planar anisotropic sheets

The *R* values were calculated using Equations 2 and 9 and were compared with measured values for fcc and bcc sheets. The slip systems $\{1\,1\,1\}$ $\langle 1\,1\,0\rangle$ and $\{1\,1\,0\}$ $\langle 1\,1\,1\rangle$ were taken for fcc and bcc metals.

Fig. 1 shows the (1 1 1) pole figure of a copper sheet [4]. The authors [4] described the texture of the sheet as composed of textures (2 2 5) $[\bar{5}\,\bar{5}\,4]$ and (1 2 3) $[1\,1\,\bar{1}]$. However, it can be described better by texture (1 1 2) $[\bar{1}\,\bar{1}\,1]$. The *R* values calculated based on texture (1 1 2) $[\bar{1}\,\bar{1}\,1]$ are in good agreement with those measured by Perovic *et al.* [4], as shown in Fig. 2.

Fig. 3 shows the (1 1 1) pole figure of a 304 stainless-steel sheet, whose texture may be approximated [6] by $\frac{1}{2}(1\,2\,3)[\bar{6}\,\bar{3}\,4] + \frac{1}{2}(\bar{1}\,2\,3)[6\,\bar{3}\,4]$.

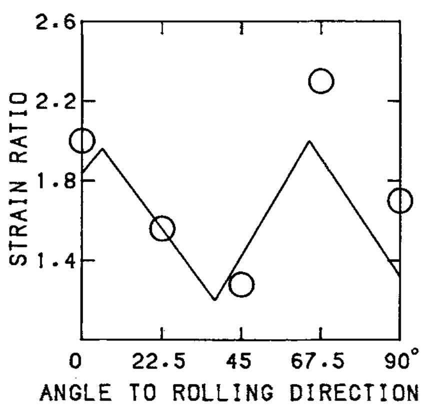

Figure 8 *R* value as a function of tensile direction for the specimen in Fig. 7: ○, measured [7]; —, calculated based on the texture (1 1 1) $[8\,\bar{9}\,1]$.

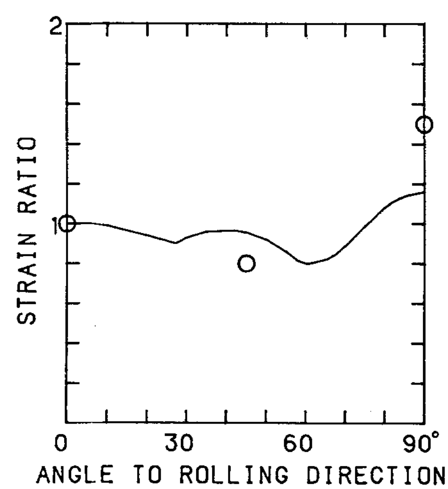

Figure 10 *R* value as a function of tensile direction for the specimen in Fig. 7: ○, measured [7]; —, calculated based on the texture $\frac{1}{2}(1\,1\,1)\,[\bar{2}\,1\,1] + \frac{1}{4}(0\,0\,1)\,[\bar{3}\,1\,0] + \frac{1}{4}(0\,0\,1)\,[\bar{1}\,3\,0]$.

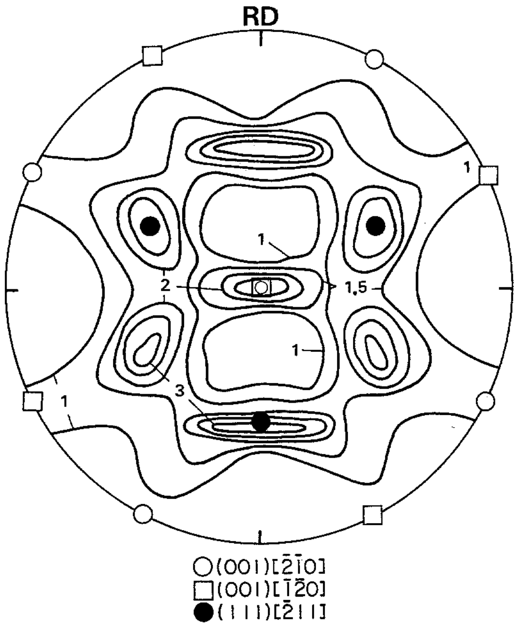

Figure 11 (0 0 1) pole figure of a 17% chromium stainless-steel sheet [8].

The R values calculated based on this texture are compared with the measured ones [6] in Fig. 4.

Fig. 5 shows the (1 1 1) pole figure of a brass sheet [4], which can be approximated by $\frac{1}{6}(1\,1\,0)$ $[\bar{1}\,1\,\bar{2}]$ + $\frac{1}{6}(1\,1\,0)$ $[\bar{1}\,1\,2]$ + $\frac{1}{3}(1\,1\,3)$ $[3\,3\,\bar{2}]$ + $\frac{1}{6}(1\,2\,3)$ $[\bar{5}\,\bar{2}\,3]$ + $\frac{1}{6}(\bar{1}\,2\,3)$ $[5\,\bar{2}\,3]$. The R values calculated from this texture are compared with the measured ones in Fig. 6.

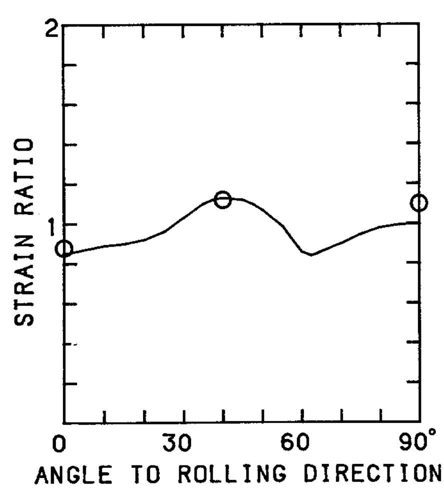

Figure 12 R value as a function of tensile direction for the specimen in Fig. 11: ○, measured [8]; —, calculated based on the texture $\frac{1}{2}(1\,1\,1)\,[\bar{2}\,1\,1]$ + $\frac{1}{4}(0\,0\,1)\,[1\,2\,0]$ + $\frac{1}{4}(0\,0\,1)$ $[2\,1\,0]$.

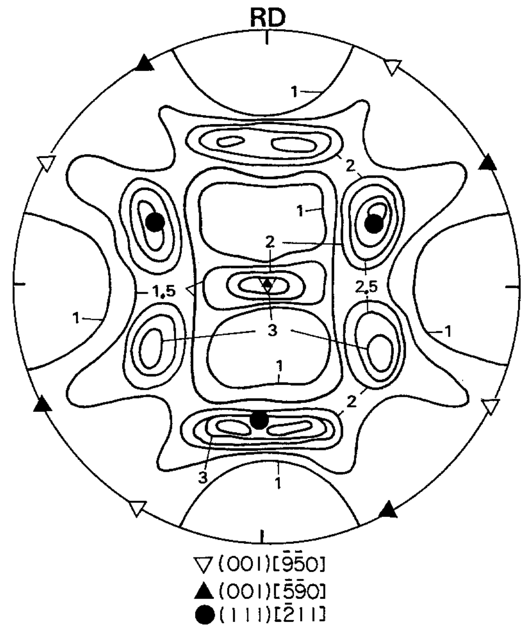

Figure 13 (0 0 1) pole figure of a 17% chromium stainless-steel sheet [8].

Fig. 7 shows the (0 0 1) pole figure of a 444 stainless-steel (b c c) sheet. The texture may be approximated by $\frac{1}{2}(1\,1\,1)\,[8\,\bar{9}\,1]$ + $\frac{1}{2}(1\,1\,1)\,[\bar{8}\,9\,\bar{1}]$, which was used to calculate the R value. The calculated values are in good agreement with those measured, as shown in Fig. 8 [7].

Fig. 9 shows the (0 0 1) pole figure of a 17% chromium stainless steel (b c c) [8], whose texture may be approximated by $\frac{1}{2}(1\,1\,1)$ $[\bar{2}\,1\,1]$ +

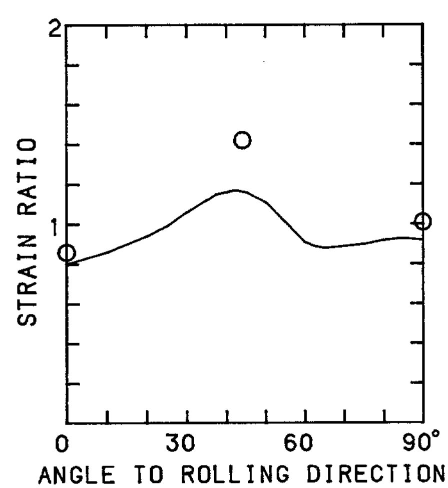

Figure 14 R value as a function of tensile direction for the specimen in Fig. 13: ○, measured [8]; —, calculated based on the texture $\frac{1}{2}(1\,1\,1)\,[\bar{2}\,1\,1]$ + $\frac{1}{4}(0\,0\,1)\,[5\,9\,0]$ + $\frac{1}{4}(0\,0\,1)$ $[9\,5\,0]$.

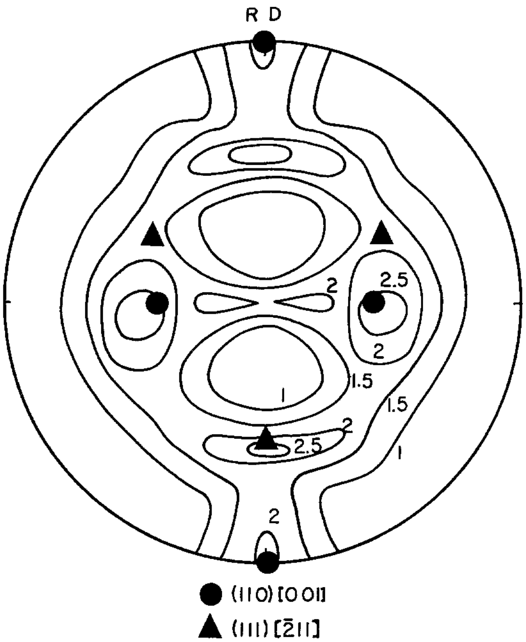

Figure 15 (0 0 1) pole figure of a 17% chromium stainless-steel sheet [8].

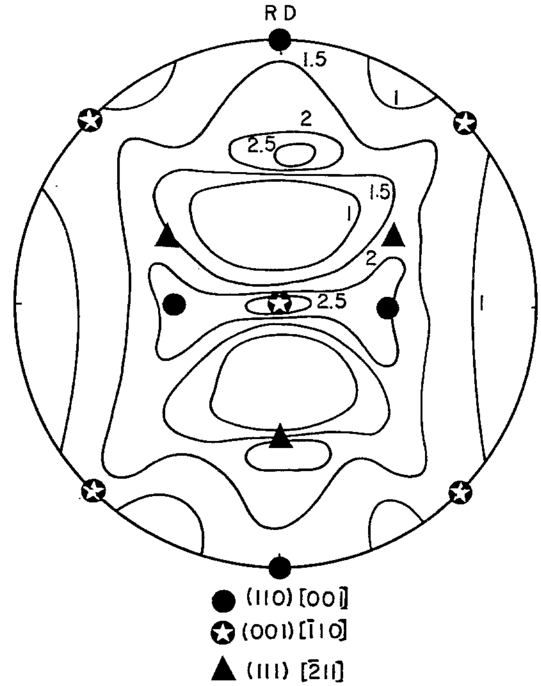

Figure 17 (0 0 1) pole figure of a 17% chromium stainless-steel sheet [8].

$\frac{1}{4}$(0 0 1) [$\bar{3}$ 1 0] + $\frac{1}{4}$(0 0 1) [$\bar{1}$ 3 0], which gives rise to the *R* values in Fig. 10.

Fig. 11 shows the (0 0 1) pole figure of a 17% chromium stainless-steel sheet [8], which may be approximated by the texture $\frac{1}{2}$(1 1 1) [$\bar{2}$ 1 1] + $\frac{1}{4}$(0 0 1) [1 2 0] + $\frac{1}{4}$(0 0 1) [2 1 0]. The *R* values of the sheet are shown in Fig. 12.

Fig. 13 shows the (0 0 1) pole figure of a 17% chromium stainless-steel sheet [8]. The texture may be described by $\frac{1}{2}$(1 1 1) [$\bar{2}$ 1 1] + $\frac{1}{4}$(0 0 1) [5 9 0] + $\frac{1}{4}$(0 0 1) [9 5 0], which yields the *R* values in Fig. 14.

Fig. 15 shows the (0 0 1) pole figure of a 17% chromium stainless-steel sheet [8]. The texture may be approximated by $\frac{1}{2}$(1 1 0) [0 0 1] + $\frac{1}{2}$(1 1 1) [$\bar{2}$ 1 1]. The *R* values of the sheet are shown in Fig. 16.

Figs. 17 and 18 are the (0 0 1) pole figure and *R* values of a 17% chromium stainless-steel sheet [8]. The *R* values were calculated based on the texture $\frac{1}{4}$(1 1 0) [0 0 1] + $\frac{1}{4}$(0 0 1) [$\bar{1}$ 1 0] + $\frac{1}{2}$(1 1 1) [$\bar{2}$ 1 1].

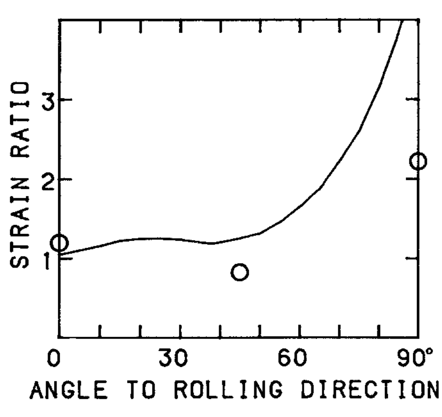

Figure 16 *R* value as a function of tensile direction for the specimen in Fig. 15: ○, measured [8]; —, calculated based on the texture $\frac{1}{2}$(1 1 0) [0 0 1] + $\frac{1}{2}$(1 1 1) [$\bar{2}$ 1 1].

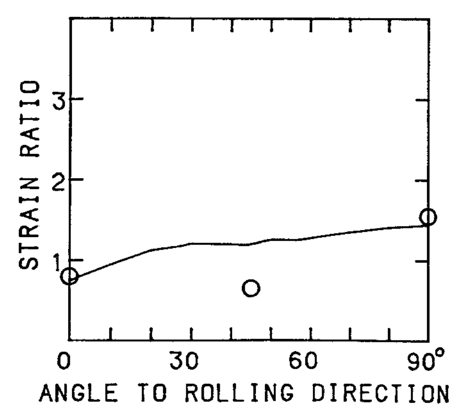

Figure 18 *R* value as a function of tensile direction for the specimen in Fig. 17: ○, measured [8]; —, calculated based on the texture $\frac{1}{4}$(1 1 0) [0 0 1] + $\frac{1}{4}$(0 0 1) [$\bar{1}$ 1 0] + $\frac{1}{2}$(1 1 1) [$\bar{2}$ 1 1].

The good agreement between measured and calculated r values in the above examples may be fortuitous, since the textures were too simplified. This method should be tested using the orientation distribution functions. However, we feel that at worst this method will be useful for a qualitative explanation of the measured R values.

5. Conclusion

The present method of calculation of strain ratio using the sheet texture gave good results. The method based on ideal textures seems to be useful for a qualitative explanation of the R values of sheets.

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